Chirality and Vorticity in Non-trivial Geometry at Finite Temperature

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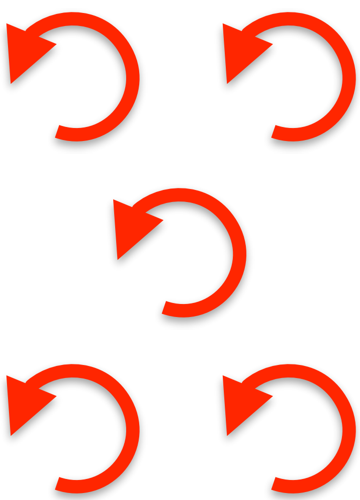
In Collaboration with Nino Flachi (Keio)
appearing in arXiv very soon (this week?)

— QCD in Finite Temperature and Heavy-Ion Collisions —

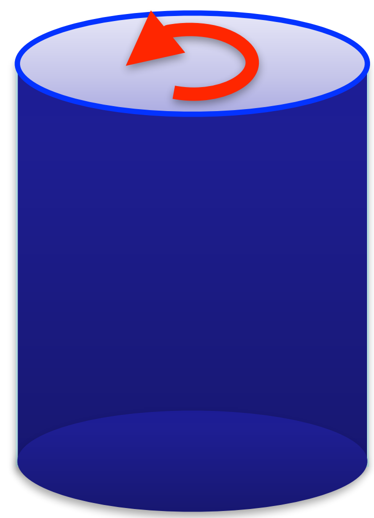
Vorticity

Fluid



$$\nabla \times \boldsymbol{u}$$

Rotating QFT



Coordinate Transformation
Finite Size (causality)

Calculations in Cylindrical Coordinates

Chen-KF-Huang-Mameda (2015)

Ebihara-KF-Mameda, 1608.00336

$$[i\gamma^\mu(\partial_\mu + \Gamma_\mu) - m]\psi = 0$$

$$g_{\mu\nu} = \begin{pmatrix} 1-(x^2+y^2)\Omega^2 & y\Omega & -x\Omega & 0 \\ y\Omega & -1 & 0 & 0 \\ -x\Omega & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Solve this in a finite cylinder (radius R)

Not only the affine connection but gamma's changed

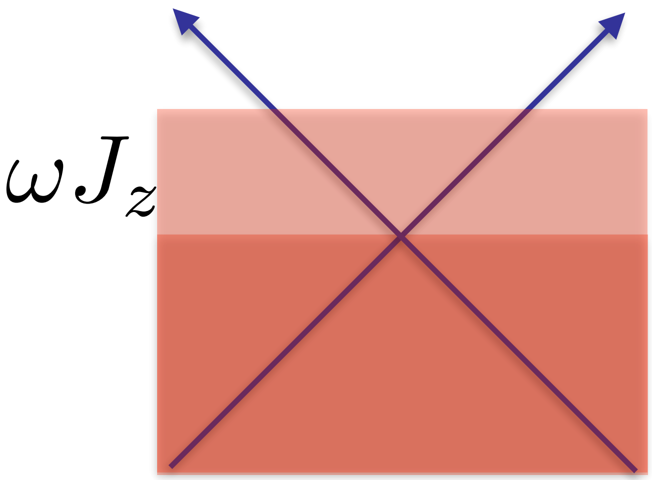
Rotation ~ B

$$B \sim \mu\omega$$

Chiral Magnetic Effect (CME) ~ Chiral Vortical Effect (CVE)

Chiral Magnetic Effect (CME)	Chiral Vortical Effect (CVE)
Gauge effect	**Geometrical effect**
Homogeneous	**Inhomogeneous** (in a rotating frame)
No upper limit	**Causality limit**
Gauge theory	**General relativity** **Fluid dynamics**

Rotation ~ Chemical Potential



Rotating fermions are given finite momenta, and the Dirac sea is "pushed up" just like chemical potentials.

Most well-known example: Deformed Nuclei

Cranking model $$H_{\mathrm{rot}} = H \underline{- \omega J_z}$$

Looks like a chemical potential for matter

cf. Fermions with rotation have the sign problem on the lattice!

No Effect in the Vacuum

$$\underbrace{\varepsilon - \Omega|\ell + 1/2|}_{\text{effective chem. pot.}} \geq \frac{1}{R}\Big[\underbrace{\xi_{\ell,1}}_{\text{smallest "mass" ~ Matsubara mode}} - \Omega R(\ell + 1/2)\Big]$$

Causality

$\Omega R \leq 1$

$$\geq \frac{1}{R}\Big[\xi_{\ell,1} - (\ell + 1/2)\Big] > 0$$

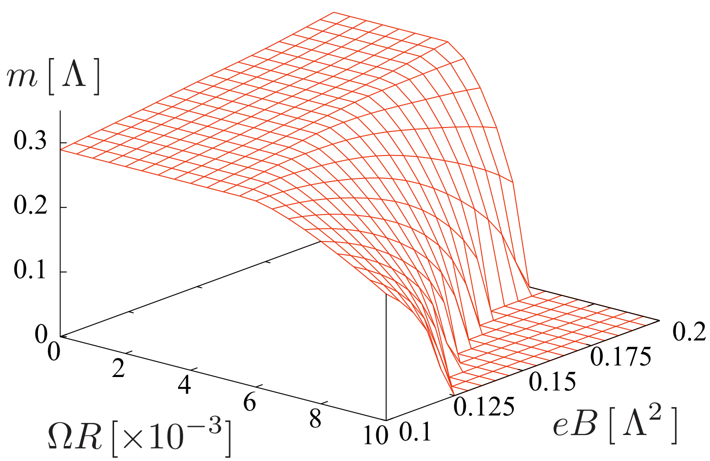
As long as "mass" is greater than "chemical potential" the vacuum remains as it is (no excitation allowed)

Anomalous effects from coupling with…

μ	B	T
Gauge CVE	**Chiral Pumping Effect**	**Gravity CVE**

Coupling to B

Chen-KF-Huang-Mameda (2015)
Chen-KF-Huang-Mameda in progress



Inverse Magnetic Catalysis
~ Finite Density System

$$n = -\frac{\partial \Omega}{\partial \mu}\bigg|_{\mu=0} = \frac{eB\omega}{4\pi^2}$$

interpreted as anomaly (**Hattori-Yin 2016**)

Can be given another interpretation as Chiral Pumping Effect

Spin Rotation (Floquet)

Starting with a Lagrangian with constant *B*

$$\psi \to \exp(\gamma^1\gamma^2\omega t/2)\psi$$

$$x \to x\cos(\omega t) - y\sin(\omega t)$$
$$y \to y\cos(\omega t) + x\sin(\omega t)$$

$$\mathcal{L} = \bar{\psi}[i\gamma^0\partial_0 + i\gamma^1(\partial_1 + ieBy/2) + i\gamma^2(\partial_2 - ieBx/2) + i\gamma^3\partial_3 + \boxed{(\omega/2)\gamma^3\gamma_5}]\psi$$

Axial Vector Field
Similar to Quarkyonic Chiral Spirals

***B*+Axial Vector Potential = Chiral Pumping Effect**

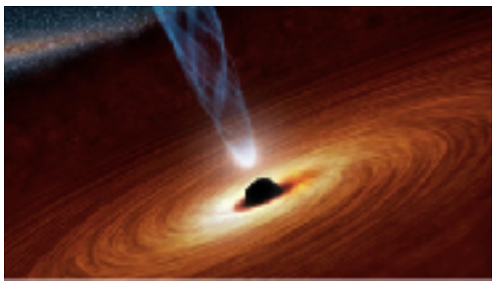
(Ebihara-KF-Oka 2015)

$$n_{\text{anomaly}} = \frac{eB\Omega}{4\pi^2}$$

Coupling to T

$T \gg R^{-1}$ **Boundary can be neglected (Debye screening)**

Vilenkin (1980)



$$\langle \vec{J}(0) \rangle = \vec{\Omega}(2\pi)^{-3} \int d^3p\, f_0'(p)$$

$$= -\vec{\Omega}\pi^{-2} \int_0^\infty f_0(p) p\, dp = -\tfrac{1}{12} \vec{\Omega} T^2$$

The most important expression in Vilenkin's paper

$$S(\vec{x}_1, \vec{x}_2, \zeta_1) = \exp\left[-i\vec{\Omega} \cdot (\vec{x}_1 \times \vec{\nabla}_1) \frac{\partial}{\partial \zeta_1} + \tfrac{1}{2}\vec{\Omega} \cdot \vec{\Sigma} \frac{\partial}{\partial \zeta_1} \right]$$

$$\times S_0(\vec{x}_1, \vec{x}_2, \zeta_1) . \qquad (54)$$

"Energy Shift Operator" **Energy Derivative**

Chiral Vortical Effect

$$S(\vec{x}_1, \vec{x}_2, \zeta_1) = \exp\left[\cancel{-i\vec{\Omega}\cdot(\vec{x}_1 \times \vec{\nabla}_1)\frac{\partial}{\partial \zeta_1}} + \tfrac{1}{2}\vec{\Omega}\cdot\vec{\Sigma}\,\frac{\partial}{\partial \zeta_1}\right] \times S_0(\vec{x}_1, \vec{x}_2, \zeta_1)\,. \tag{54}$$

$$\langle j_5^\mu \rangle \sim \langle \mathrm{tr}[\gamma^\mu \gamma_5 S(x,x)] \rangle \qquad \sim \mathrm{tr}[\gamma_5 \gamma^\mu \gamma^\alpha \gamma^\beta \gamma^\nu]$$

$$\int \frac{d^4 p}{(2\pi)^4}$$

$$\frac{\gamma_\nu p^\nu + m}{p^2 - m^2}$$

$$\sim \mathbf{\Omega}\cdot\mathbf{\Sigma}\,\frac{\partial}{\partial p_0}$$

Surface term!
Vanishing at *T*=0!

"Anomalous" current present at finite *T* !

Mixed Gravitational Anomaly

Landsteiner-Megias-Pena-Benitez (2011)

$$\nabla_\mu j_A^\mu = C_F \epsilon^{\mu\nu\rho\lambda} F_{\mu\nu} F_{\rho\lambda} + C_R \epsilon^{\mu\nu\rho\lambda} R_{\mu\nu}{}^{\alpha\beta} R_{\rho\lambda\alpha\beta}$$

$$\text{CVE } J_A^\mu \sim \quad \omega\mu^2 \qquad\qquad \omega T^2$$

CME can be understood from the Chern-Simons current:

$J_A^\mu - 4C_F \epsilon^{\mu\nu\rho\lambda} A_\nu \partial_\rho A_\lambda$ is a conserved current

Chern-Simons current is zero for physical states

$A_0 \leftrightarrow \mu$ **CS current can be finite (CME current ~ μB)**

How to derive the CVE from the gravitational CS current?

Gravitational CS Current

$$J_A^\mu = 4C_R \epsilon^{\mu\nu\rho\lambda}\Gamma^\alpha_{\nu\beta}\left(\partial_\rho \Gamma^\beta_{\alpha\lambda} + \frac{2}{3}\Gamma^\beta_{\rho\sigma}\Gamma^\sigma_{\alpha\lambda}\right)$$

You may think that this is NOT "gauge" invariant because the Christoffel symbols are NOT tensors

BUT!

What is vorticity at all?

$\omega + \text{coordinate rotation} \;\to\; \omega + \delta\omega$

This is impossible for vectors or tensors (zero is always zero)

$\omega^z = 2\omega_{xy} = {\Gamma^x}_{0y} = -{\Gamma^y}_{0x}$ cf. Coriolis force

CS current ~ $J_A^\mu \;\sim\; \omega R$ ***T*** **^{2} where???**

Direct Calculation with R and ω

Flachi-KF (appearing this week)

$$S(\boldsymbol{x},\boldsymbol{x}',k_0) = e^{\boldsymbol{\omega}\cdot\frac{1}{2}\boldsymbol{\Sigma}\frac{\partial}{\partial k_0}} S_0(\boldsymbol{x},\boldsymbol{x}',k_0)$$ **(in the normal coordinates)**

$$S_0(x,x') = \int \frac{d^D k}{(2\pi)^D}\,(-\gamma^\mu k_\mu + m)\,\mathcal{G}(k)$$

$$\mathcal{G}(k) = \left[1 - \left(A_1(x') + iA_{1\alpha}(x')\frac{\partial}{\partial k_\alpha} - A_{1\alpha\beta}(x')\frac{\partial^2}{\partial k_\alpha \partial k_\beta}\right)\frac{\partial}{\partial m^2} + A_2(x')\left(\frac{\partial}{\partial m^2}\right)^2\right]\frac{1}{k^2 - m^2}$$

$$j_A^\mu = i\,\epsilon^{\mu\mu'\nu'\nu}\omega_{\mu'\nu'}\int \frac{d^4k}{(2\pi)^4}\,\frac{\partial}{\partial k_0}\,k_\nu\,\mathcal{G}(k)$$

up to the 1st order in ω

Direct Calculation with R and ω

Flachi-KF (appearing this week)

$$j_A^\mu = i\,\epsilon^{\mu\mu'\nu'\nu}\omega_{\mu'\nu'} \int \frac{d^4k}{(2\pi)^4}\,\frac{\partial}{\partial k_0}\,k_\nu\,\mathcal{G}(k)$$

Leading-order contribution ~

$$\int \frac{d^4k}{(2\pi)^4}\,\frac{\partial}{\partial k_0}\,\frac{k_0}{k^2 - m^2} = -i\int \frac{d^3k}{(2\pi)^3}\,n_F'(\varepsilon_k)$$

$$= \frac{i}{\pi^2}\int_0^\infty dk \left(\varepsilon_k - \frac{m^2}{2\varepsilon_k}\right) n_F(\varepsilon_k) \longrightarrow \frac{i}{12}T^2$$

Direct Calculation with R and ω

Flachi-KF (appearing this week)

$$j_A^\mu = i\,\epsilon^{\mu\mu'\nu'\nu}\omega_{\mu'\nu'} \int \frac{d^4k}{(2\pi)^4}\,\frac{\partial}{\partial k_0}\,k_\nu\,\mathcal{G}(k)$$

Next to leading-order contribution ~

$$A_1 = \frac{R}{12} \times$$

$$\frac{\partial}{\partial m^2}\int \frac{d^4k}{(2\pi)^4}\,\frac{\partial}{\partial k_0}\,\frac{k_0}{k^2-m^2} = -\frac{i}{2}\int\frac{d^3k}{(2\pi)^3}\,\varepsilon_k^{-1} n_F''(\varepsilon_k)$$

(no IR singularity) $\longrightarrow$ $-\dfrac{i}{96\pi^2}R$

Direct Calculation with R and ω

Flachi-KF (appearing this week)

$$j_A^\mu = i\,\epsilon^{\mu\mu'\nu'\nu}\omega_{\mu'\nu'}\int\frac{d^4k}{(2\pi)^4}\frac{\partial}{\partial k_0}k_\nu\,\mathcal{G}(k)$$

$$j_A^z = \omega\left(\frac{T^2}{12} - \frac{m^2}{8\pi^2} - \frac{R}{96\pi^2} + \cdots\right)$$

first correction by finite mass

related through "Chiral Gap Effect"

$$m^2 \;\to\; m^2 + \frac{R}{12}$$

Flachi-Fukushima PRL(2014)

Finite-*T* CVE and CS current connected by the mass term!

CS Current ~ Physical Current

For the gravitational CS current,
no reason why its expectation value should be zero

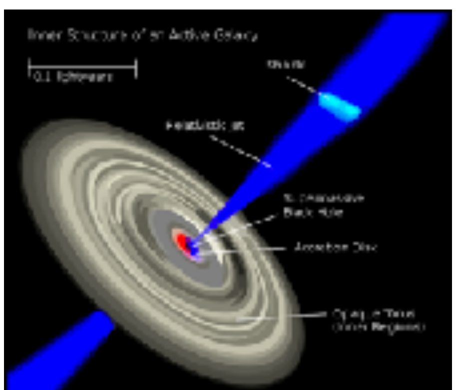
CS current on Kerr geometry (Boyer-Lindquist coordinates)

Up to the linear order in ω

$$\times 1/96\pi^2$$

$$j^r_{\rm CS} = \frac{3\pi^3(-3\pi + 8rT_B)\chi}{256 r^6 T_B^4}\omega\,, \qquad j^\chi_{\rm CS} = \frac{3\pi^3(-1+3\chi^2)}{64 r^6 T_B^3}\omega$$

$$\chi = \cos\theta$$



may be an origin for "astrophysical jet"
in the universe (~ particle production)

(Wikipedia)

CS Current ~ Physical Current

For the gravitational CS current,
no reason why its expectation value should be zero

CS current on Kerr geometry (Boyer-Lindquist coordinates)

Another interesting limit ~ Extremal limit (zero temperature)

$$j^r_{\mathrm{CS}} = -\frac{32(1-2\xi)[\chi^4 + 4\chi^2\xi(3-8\xi) - 48\xi^3(1-\xi)]\chi}{(\chi^2+4\xi^2)^5}\omega^3$$

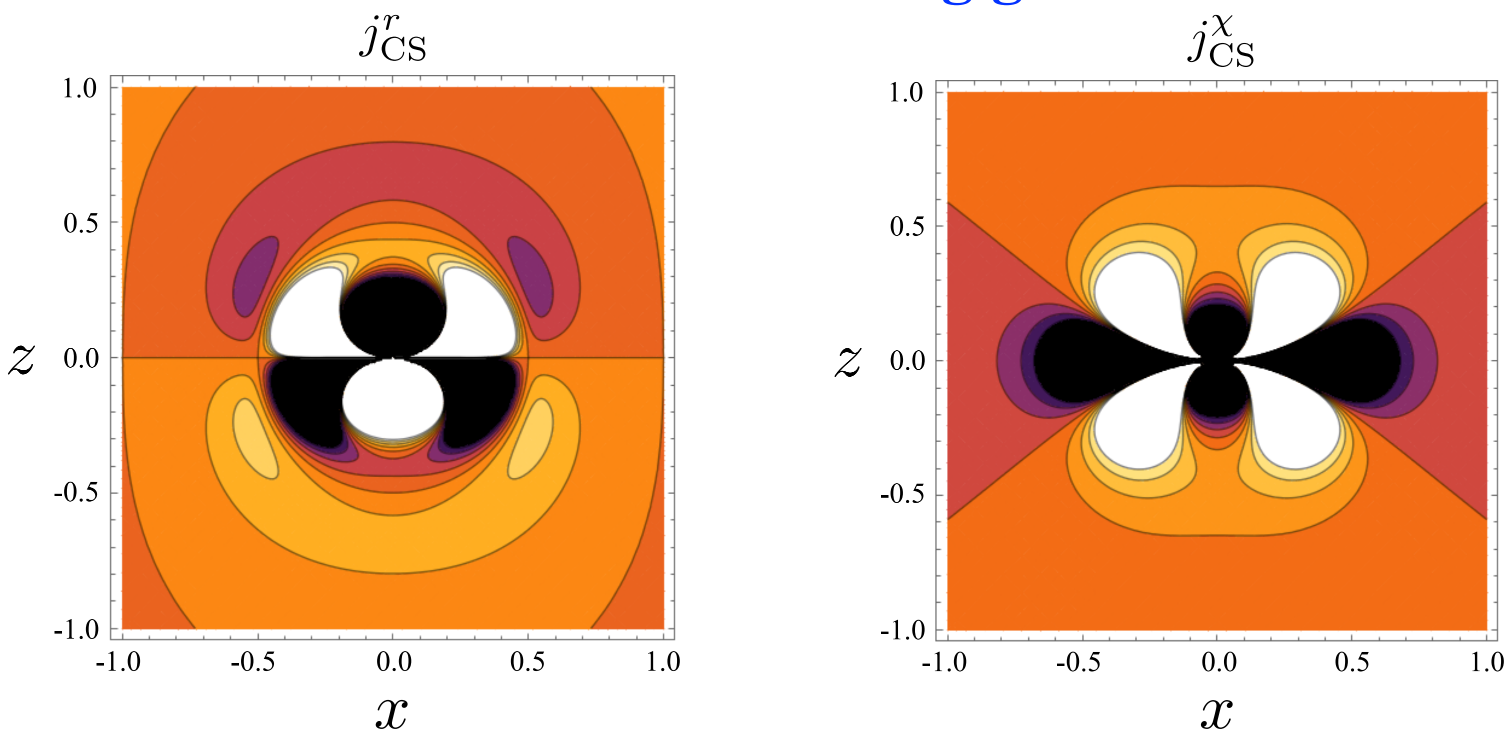
$$j^\chi_{\mathrm{CS}} = \frac{32[\chi^6 - \chi^4(3+56\xi^2) + 72\chi^2\xi^2(1+2\xi^2) - 48\xi^4]}{(\chi^2+4\xi^2)^5}\omega^4$$

$$\xi = r\omega$$

Small ω and small T_{B} are NOT commutable limits

CS Current ~ Physical Current

Distribution of currents on rotating gravitational background



Big enhancement at $\chi = 0$ ($z = 0$) → jet + disk!

Summary

- CS current is a physical current if non-zero seen with physical states
- Standard CVE formula and the gravitational CS current connected through the finite mass correction and the chiral gap effect
- CS current provides us with a non-perturbative device to obtain a physical current for general geometrical backgrounds

- **Effects of rotation deserve further investigations!**